SHORT ANSWER PROBLEMS

(1) Alex and Benito make 880 pies in 8 hours working together. Alex makes 10 more pies in one hour than Benito. Find the number of pies made by Alex in one hour. 【**Submitted by Philippines**】

【Solution】

In 8 hours Alex makes $10 \times 8 = 80$ more pies than Benito. Hence if Benito made same pies as Alex in one hour, they can make $880 + 80 = 960$ pies in 8 hours. So Alex made $960 \div 8 \div 2 = 60$ pies per hour.

***Answer:* 60 pies**

(2) Divide 108 students into four groups such that two times the number of students in group 1 is

(i) half of the number of students in group 2,

(ii) 2 less than the number of students in group 3.

(iii) 2 more than the number of students in group 4.

Find the number of students in group 1. 【**Submitted by Philippines**】

【Solution】

If we move two students from Group Three to Group Four, then each of them has twice as many students as Group One, while Group Two has four times as many. Hence there are $108 \div (1+4+2+2) = 12$ students in Group One.

***Answer:* 12 students**

(3) In the diagram below, *C*, *D* and *E* are points on the line *AB*. Given *AB* = 9.2 cm and *CE* = 4.7 cm, find the sum of the lengths of all ten line segments determined by these five points. 【**Submitted by Thailand**】

【Solution】

There are 10 line segments, the sum of lengths of which is:

$AB + (AC + CB) + (AD + DB) + (AE + EB) + CE + (CD + DE) = 4 \times 9.2 + 2 \times 4.7 = 46.2$ cm.

***Answer*: 46.2 cm**

(4) Four cube with edge length 1 m are cut up into cubes each with edge length 4 cm. If all these cubes were placed one on the right of the other to form a line, find the length of the line, in m. 【**Submitted by Jury**】

【Solution 1】

Since 1 m = 100 cm, there are $100 \div 4 = 25$ of the 4 cm cubes along each edge of the 1 m cube. So there are $25 \times 25 \times 25 \times 4$ of the 4 cm cubes. So, when they are placed one on the right of the other, the height is $25 \times 25 \times 25 \times 4 \times 4 = 250000$ cm = 2500 m.

【Solution 2】

Observe that the sum of the volumes of the cubes with length 4 cm is equal to the sum of the volumes of the four cubes with length 1 m, which is 4 m^3 = 4000000 cm^3. Since the area of each face of a cube with length 4 cm is $4\times4=16\text{cm}^2$, the length of the line is $4000000\div16=250000\text{cm}$ = 2500 m.

***Answer*: 2500 m**

(5) Michael wanted to tie 20 ropes. The length of each rope was 50 cm. 5 cm of one end of a rope was tied to 5 cm of one end of another rope. Each of the resulting knots was 5 cm long. What was the length, in cm, of the new rope?

【Submitted by Brunei】

【Solution】

Since there were 20 ropes, the total length of these ropes was $20\times50=1000\text{cm}$. However, some parts of the ropes were used to tie the knots. So the new rope was shorter than the total length of these ropes.

Since there were 20 pieces of ropes altogether and the two ends of the rope were not tied, the number of knots tied was $20-1=19$.

$5+5=10\text{cm}$ of rope was used to tie each knot but the knot was only 5 cm long. Thus, the new rope would be $19\times5=95\text{cm}$ shorter than the total length of the 20 ropes. Therefore, the new rope was $1000-95=905\text{cm}$ long.

***Answer*: 905 cm**

(6) Class A and Class B have the same number of students.

- The number of students in class A who took part in a mathematics competition is $\frac{1}{3}$ of the students in class B who did not take part.
- The number of students in class B who took part in a mathematics competition is $\frac{1}{5}$ of the students in class A who did not take part.

Find the ratio of the number of students in class A who did not take part in this competition to the number of students in class B who did not take part.

【Submitted by Malaysia】

【Solution】

Let a and b be the number did not take part in class A and B respectively. Then the number of students in class A is $a+\frac{1}{3}b$, the number of students in class B is $b+\frac{1}{5}a$.

Since Class A and B have the same number of students, we have $a+\frac{1}{3}b=b+\frac{1}{5}a$,

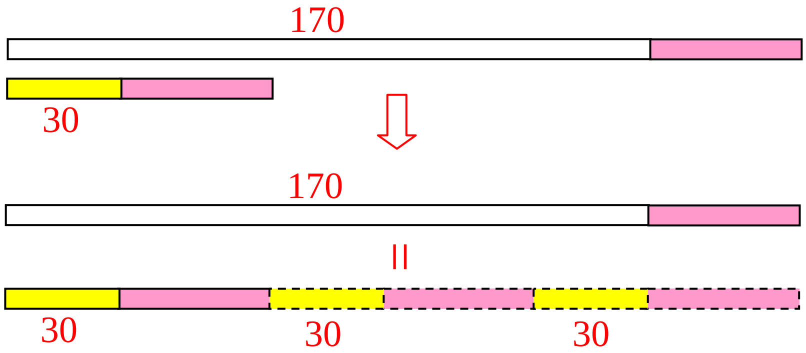
and hence $\frac{4}{5}a=\frac{2}{3}b$, i.e. $a : b = 5 : 6$.

***Answer*: 5 : 6**

(7) What number can be added to both 170 and 30 so that the sums are in the ratio 3: 1? 【**Submitted by Philippines**】

【Solution】

Note that $170-3\times30=80$. The unknown number is added to 90 three times but added to 170 only once. So twice this number is 80 and this number is 40.



***Answer:* 40**

(8) Two different shirts at a shop were sold at the same price. While one shirt made a profit of 30%, the shop had incurred a 30% loss for the other one. Did the shop record a profit or loss from these two transactions, and by how many %? 【**Submitted by Thailand**】

【Solution】

Suppose each shirt is sold for $(100+30)(100-30)=9100$ dollars. Then the cost of the first shirt is $9100\div(100\%+30\%)=7000$ dollars and the cost of the second shirt is $9100\div(100\%-30\%)=13000$ dollars. The total cost is 20000 dollars, and the total take is $9100\times2=18200$ dollars. Hence the net loss is $\frac{1800}{20000}=\frac{9}{100}=9\%$.

***Answer*: loss 9 %**

(9) A television show has 483 episodes. If the show starts on Saturday and broadcasts everyday with three episodes each day, on what day will the last episode be broadcasted? 【**Submitted by Philippines**】

【Solution】

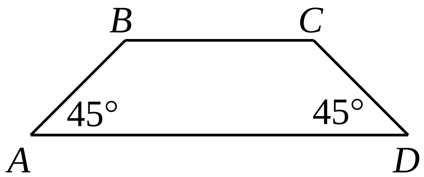
Since there are three episodes broadcasted each day, it will take $483\div3=161$ days to broadcast 483 episodes. Since Saturday is counted as the first day, Sunday will be counted as the second day, and so on:

Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
1	2	3	4	5	6	7

$161=23\times7$, so the last episode will be broadcasted on Friday.

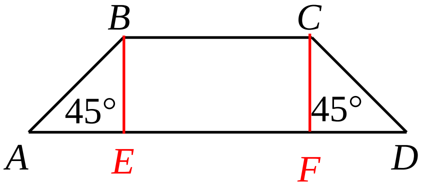
***Answer*: Friday**

(10) Find the area, in cm^2, of the isosceles trapezoid $ABCD$, given that $AD=16$ cm, $BC=8$ cm, $AB=CD$ and $\angle A=\angle D=45°$. 【**Submitted by Vietnam**】



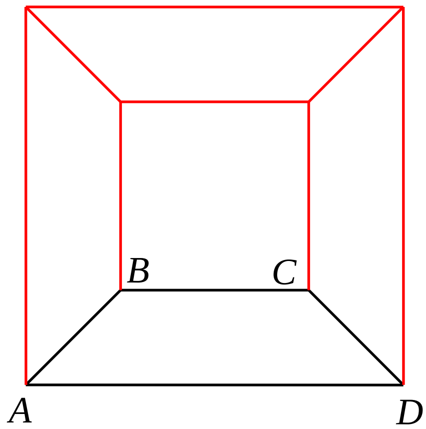
【Solution 1】

Let E and F be the points on AD so that $BE \perp AD$ and $CF \perp AD$, respectively, as the diagram shown below.



Observe that $BE = CF$ and $EF = BC = 8$ cm. Since $\angle A = \angle D = 45°$, $BE \perp AD$ and $CF \perp AD$, we know $\angle ABE = \angle DCF = 45°$ and hence $\triangle ABE \cong \triangle DFC$. So $AE = DF = \frac{1}{2}(16-8) = 4$ cm. Thus $BE = CF = 4$ cm and we can conclude that the area of isosceles trapezoid $ABCD$ is $\frac{(16+8)\times 4}{2} = 48\text{ cm}^2$.

【Solution 2】



Construct 3 additional trapezoids identical to trapezoid $ABCD$ to create a large square, inside which there is a small square. Then the area of trapezoid $ABCD$ is one fourth the difference between the two squares and is equal to

$$(16^2 - 8^2) \div 4 = (256-64) \div 4 = 48\text{ cm}^2.$$

Answer: **48 cm^2**

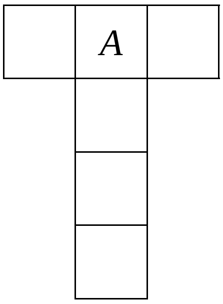
(11) On her 40th birthday, Mrs. Sharma makes gifts to her two sons whose ages are prime numbers. She gives to one son a number of dollars equal to the square of his age, and to the other son a number of dollars equal to his age. She gives 300 dollars in total. Find the sum of the ages of Mrs. Sharma's two sons. **【Submitted by India】**

【Solution】

Since $289 = 17^2 < 300 < 19^2 = 361$, hence must have a son younger than 19.
If a son's age is 17, then another son's age is $300-289=11$, which is also a prime.
If a son's age is less than 17, then another son's age is no less than $300-169=131$, which is impossible since the ages of his son must be less than the mother's.
So the sum of the ages of Mrs. Sharma's two sons is $17+11=28$.

Answer: **28**

(12) The numbers 5, 6, 7, 8, 9, 10 are to be filled in the squares so that the sum of the numbers in the row is equal to the sum of the numbers in the column. How many different possible values of *A* are there? **【Submitted by Malaysia】**



【Solution】

The sum of the two numbers in the row other than *A* is equal to the sum of the three numbers in the column other than *A*. We have $10+9=8+6+5$ and $10+8=7+6+5$. Already $10+7<8+6+5$. Hence there are only two possible cases, with $A=7$ in the first and $A=9$ in the second.

***Answer:* 2**

(13) A farmer harvested 2016 apples. He wishes to pack them as many boxes as possible, not necessarily packing all the apples, with each box a whole number of apples. The second box must be 10 more than the first, the third 10 more than the second and so on. What is the smallest number of apples left unpacked? **【Submitted by Jury】**

【Solution】

Start with a first box of 1 apple. The numbers will be 1, 11, 21, and so on, and we are looking for a sum of numbers to be less than 2016. The sum is $1+11+21+31+\cdots+181+191=1920$ and there are 20 boxes. This leaves 96 apples to be divided as much as possible into 20 equal parts which can be added to each box. The largest such number is the integer part of $96\div20$ which is 4. So if we start with a box with 5 apple, we get the sum of the numbers $5+15+25+35+\cdots+185+195=2000$. So the smallest number of apples left is 16.

***Answer*: 16 apples**

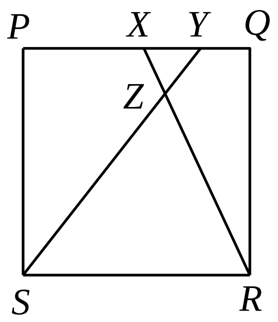
(14) Three containers A, B, and C contain a total of 48 apples. First, 6 apples are taken from A and are put into B. Second, 9 apples are taken from B and are put into C. Now, each container has the same number of apples. What is the original number of apples in container A? **【Submitted by Philippines】**

【Solution】

Since there are a total of 48 apples and each container has the same number of apples now, each container has $48\div3=16$ apples. Clearly there were $16+6=22$ apples in container A.

***Answer:* 22 apples**

(15) The square *PQRS* has area of 400 cm^2. The points *X* and *Y* divide *PQ* into 3 parts.



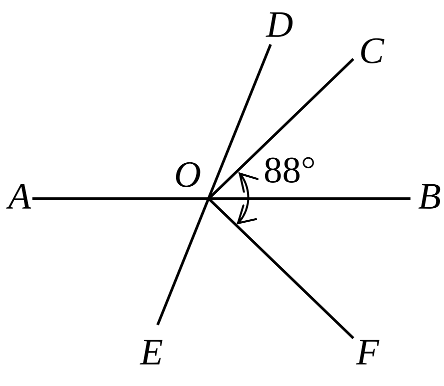
If the perimeter of triangle *XYZ* is $\frac{1}{4}$ of the perimeter of triangle *SRZ*, find the area, in cm^2, of $\triangle XYZ$. **【Submitted by Jury】**

【Solution】

The triangles *XYZ* and *RSZ* are similar. Since $400 = 20^2$, the side length of square *PQRS* is 20 cm. As the perimeter of triangle *XYZ* is $\frac{1}{4}$ of the perimeter of triangle *SRZ*, $XY = \frac{1}{4} \times SR = 5$cm and the height of $\triangle XYZ$ is $\frac{1}{4}$ of the height of $\triangle SRZ$ or $\frac{1}{5}$ of the side of the square, so the area of $\triangle XYZ$ is $\frac{1}{2} \times 5 \times \frac{1}{5} \times 20 = 10$ cm^2.

***Answer*: 10 cm^2**

(16) In the diagram, line *AB* and line *DE* meet in *O* and $\angle COF = 88°$. Given that *OE* is the angle bisector of $\angle AOF$ and *OB* is the angle bisector of $\angle COF$. Find the measure, in degrees, of $\angle COD$. **【Submitted by Philippines】**



【Solution】

Since *OB* is the angle bisector of $\angle COF$, we have $\angle COB = \angle BOF = 44°$ and $\angle AOF = 180° - 44° = 136°$.

Since *OE* is the angle bisector of $\angle AOF$, we have $\angle EOF = \frac{136°}{2} = 68°$.

Hence $\angle COD = 180° - 88° - 68° = 24°$.

Answer: 24°

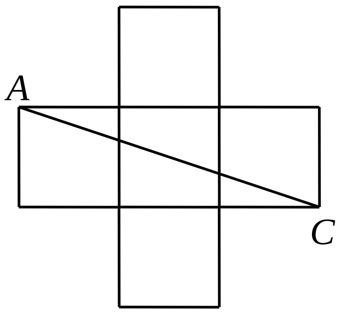
(17) $\overline{243a688} \div \overline{31b2} = 764$, find the value for $a \times b$. **【Submitted by Indonesia】**

【Solution】

When 2430688 is divided by 764, the remainder is 404. Therefore, a404 is a multiple of 764, and we have $8404 = 11 \times 764$. Hence $a = 8$. Dividing 2438688 by 764, the quotient is 3192. Hence $b = 9$ and $a \times b = 72$.

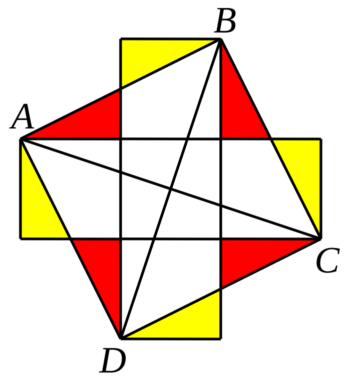
***Answer*: 72**

(18) Find the area of the cross made of five identical squares in the figure below, given that the length of AC is 12 cm. **【Submitted by Vietnam】**



【Solution 1】

Construct additional lines and replace the 4 yellow right triangles with the 4 red right triangles.



Then the area of the cross equals the area of square $ABCD$ which is $\frac{1}{2}AC^2 = 72\,\text{cm}^2$.

【Solution 2】

Let the side length of the squares is a, then the area of the cross is $5a^2$.

From Pythagoras Theorem, we have $(3a)^2 + a^2 = 10a^2 = 12^2 = 144$, hence the area of the cross is $5a^2 = \frac{1}{2} \times 144 = 72\,\text{cm}^2$.

***Answer:* 72 cm^2**

(19) Three positive two-digit integers and 63 are arranged in a 2×2 table. For each row and column of the table, the product of the two numbers in this row or column is calculated. When all four such products are added together, the result is 2016. What is the largest possible number in the square A of the table? **【Submitted by Jury】**

A	
	63

【Solution】

Let the three positive two-digit integers be *a*, *b* and *c* as shown.

a	*b*
c	63

We have $ab+63c+ac+63b=2016$, so $(a+63)(b+c)=2016=2^5\times3^2\times7$.

Since $b+c\geq20$, the smallest possible value of $b+c$ is $7\times3=21$. Thus the largest possible value of $a+63$ is $2^5\times3=96$, i.e. the largest value of *a* is $96-63=33$.

An example as following:

33	11
10	63

***Answer*: 33**

(20) Ali has 5 consecutive numbers while Ben has 7 consecutive numbers, none of the Ali's number is in the group of Ben's numbers. If the second number of Ali's number is 5 and the sum of Ali's and Ben's numbers are 128. What is the largest number of Ben's number? **【Submitted by Indonesia】**

【Solution】

If the second number of Ali's number is 5, so the Ali's number are 4, 5, 6, 7, 8 and their sum is 30. From this we got the sum of Ben's number is $128-30=98$.

Since Ben has 7 consecutive numbers, hence its middle number is $98\div7=14$, and the largest number is $14+3=17$.

***Answer:* 17**

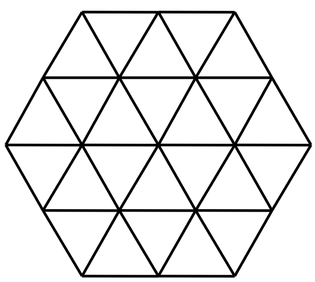
(21) Sam, Tom and Una are three chefs of a restaurant. One day, they cooked 320 plates of spaghetti and in this day, Sam cooked for 6 hours, Tom cooked for 8 hours and Una cooked for 5 hours. They also cook spaghetti at different speeds, with Sam cooking 5 plates for every 3 plates Tom cooks and every 2 plates Una cooks. How many plates of spaghetti did Sam cook this day? **【Submitted by Jury】**

【Solution】

If we multiply the rates of work by the days worked for each chef, we get the ratio of their effective total workload. Sam : Tom : Una = 30 : 24 : 10. So for every 64 plates, Sam cooks 30, Tom cooks 24 and Una cooks 10. $320\div64=5$, so Sam cooks $5\times30=150$ plates of spaghetti.

***Answer*: 150 plates**

(22) How many equilateral triangles are in the figure below, in all possible sizes and directions? **【Submitted by Vietnam】**



【Solution】

The number of triangles with area equal 1 is: 24.
The number of triangles with area equal 4 is: 12.
The number of triangles with area equal 9 is: 2.
Hence the total number of triangles is 24+12+2=38.

***ANS*:38 equilateral triangles**

(23) With the appropriate order of the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9, find the smallest 9-digit number that is divisible by 99.【**Submitted by Vietnam**】

【Solution 1】

Let the number formed be $\overline{a_1a_2a_3a_4a_5a_6a_7a_8a_9}$, where $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ is a permutation of (1, 2, 3, 4, 5, 6, 7, 8, 9).

Because $a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9=1+2+3+4+5+6+7+8+9=45$ is divisible by 9, $\overline{a_1a_2a_3a_4a_5a_6a_7a_8a_9}$ is divisible by 9. In order for $\overline{a_1a_2a_3a_4a_5a_6a_7a_8a_9}$ to be divisible by 99, the following difference must be divisible by 11:

$$(a_1+a_3+a_5+a_7+a_9)-(a_2+a_4+a_6+a_8)=45-2(a_2+a_4+a_6+a_8)$$

In order for the above difference to be divisible by 11, $a_2+a_4+a_6+a_8$ must leave a remainder of 6 when divided by 11.

Since $10 \le a_2+a_4+a_6+a_8 \le 30$, $a_2+a_4+a_6+a_8$ equals 17 or 28.

(a) Consider $a_2+a_4+a_6+a_8=28$, then a_2, a_4, a_6, $a_8 \ge 4$.
In order for $\overline{a_1a_2a_3a_4a_5a_6a_7a_8a_9}$ to be as small as possible, let $a_1=1$, $a_2=4$, and the smallest number possible is 142738596.

(b) Consider $a_2+a_4+a_6+a_8=17$.
In order for $\overline{a_1a_2a_3a_4a_5a_6a_7a_8a_9}$ to be as small as possible, let $a_1=1$.
Then (a_2, a_4, a_6, a_8) is a permutation of (2, 3, 4, 8) or (2, 4, 5, 6).
Thus, the smallest number possible in this case is 123475869.

From (a) and (b), we have the smallest number that can be formed is 123475869.

【Solution 2】

Let the number formed be $\overline{a_1a_2a_3a_4a_5a_6a_7a_8a_9}$, where $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ is a permutation of (1, 2, 3, 4, 5, 6, 7, 8, 9).

Because $a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9=45$, $\overline{a_1a_2a_3a_4a_5a_6a_7a_8a_9}$ is divisible by 9. In order for $\overline{a_1a_2a_3a_4a_5a_6a_7a_8a_9}$ to be divisible by 99, the following difference must be divisible by 11:

$$D=(a_1+a_3+a_5+a_7+a_9)-(a_2+a_4+a_6+a_8)$$

Observe that $D \ne 0$ since $a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9=45$. Since we want to find the smallest 9-digit number, we can take $a_1=1$ first.

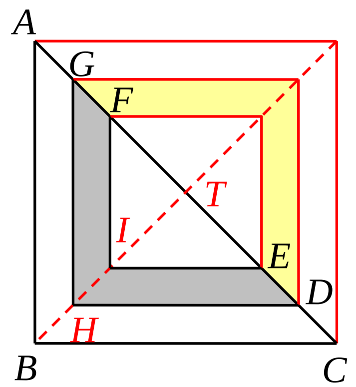
Then $D \le (1+6+7+8+9)-(2+3+4+5)=17$.

So $D=11$ and hence $a_2+a_4+a_6+a_8=\dfrac{45-11}{2}=17$. Now we can try $a_2=2$, $a_3=3$ and $a_4=4$. Thus $a_6+a_8=17-6=11$. Since all of 1, 2, 3 and 4 can't be the value of a_6, the possible values of a_6 are 5 and 6. Thus we can take $a_6 = 5$ and hence $a_8 = 6$. Now $a_5 + a_7 + a_9 = 24$ and the sum of unused digits is $7 + 8 + 9 = 24$, so we can take $a_5 = 7$, $a_7 = 8$ and $a_9 = 9$ to get the smallest number that can be formed is 123475869.

***Answer:* 123475869**

(24) In the diagram shown below, *ABC*, *DGH* and *EFI* are isosceles right triangles. Given $AG = GF = CD = DE = 1$cm and $FE = 4$cm. Find the ratio of area of shaded region to the area of triangle *ABC*. 【**Submitted by Indonesia**】

【Solution】



Note that $AC = 8$cm and $DG = 6$cm.

Make another picture and combine it so we can make a square, and point *T* is the intersection of diagonal of the square, as the figure shown. Now we can conclude:

Two times of the area of shaded region equal to the different of the area of middle square and smallest square, that is $6 \times 6 \div 2 - 4 \times 4 \div 2 = 10\text{cm}^2$.

Hence the ratio of area of shaded region to the area of triangle *ABC* is $\dfrac{10}{8 \times 8 \div 2} = \dfrac{5}{16}$.

Answer: $\dfrac{5}{16}$

(25) Whenever Sam reads a date like 20/11/2016, he incorrectly interprets it as two divisions, with the second one evaluated before the first one:

$$20 \div (11 \div 2016) = \frac{40320}{11} = 3665\frac{5}{11}$$

For some dates, like this one, he does not get an integer, while for others, like 20/8/2016, he gets $20 \div (8 \div 2016) = 5040$, an integer. How many dates this year (day/month/year) give him a non-integer? 【**Submitted by Jury**】

【Solution】

A date *d*/*m*/2016 will give the fraction $d \div \dfrac{m}{2016} = \dfrac{2016d}{m}$ which is a whole number whenever 2016*d* is a multiple of *m*. Now, $2016 = 2^5 \times 3^2 \times 7$, so all days in months 1, 2, 3, 4, 6, 7, 8, 9 and 12 give integers, and in months 10, the number *d* needs to be a multiple of 5. In all other months, *d* needs to be a multiple of *m*. In summary:

month *m*	integer days *d*	number of integer days
5	5, 10, 15, 20, 25, 30	6
10	5, 10, 15, 20, 25, 30	6
11	11, 22	2

Totally $(31 - 6) + (31 - 6) + (30 - 2) = 78$ dates give him a non-integer.

***Answer*: 78 dates**